COURSEWORK

IMPERIAL COLLEGE LONDON

DEPARTMENT OF COMPUTING

Title of course

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Figure 1: This is a figure.

Table 1: Notation

Scalars	x
Vectors	$\boldsymbol{x}$
Matrices	$\boldsymbol{X}$
Transpose	$^\top$
Inverse	$^{-1}$
Real numbers	$\mathbb{R}$
Expected values	$\mathbb{E}$

1 Introduction

This is a template for coursework submission. Many macros and definitions can be found in `notation.tex`. This document is not an introduction to LaTeX. General advice if get stuck: Use your favorite search engine. A great source is also `https://en.wikibooks.org/wiki/LaTeX`.

2 Basics

2.1 Figures

A figure can be included as follows: Fig. 1 shows the Imperial College logo.
Some guidelines:

- Always use vector graphics (scale free)
- In graphs, label the axes
- Make sure the font size (labels, axes) is sufficiently large
- When using colors, avoid red and green together (color blindness)
- Use different line styles (solid, dashed, dotted etc.) and different markers to make it easier to distinguish between lines

2.2 Notation

Table 1 lists some notation with some useful shortcuts (see latex source code).

2.2.1 Equations

Here are a few guidelines regarding equations

- Please use the `align` environment for equations (`eqnarray` is buggy)
- Please number all equations: It will make things easier when we need to refer to equation numbers. If you always use the `align` environment, equations are numbered by default.
- Vectors are by default column vectors, and we write

$$\boldsymbol{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \tag{1}$$

- Note that the same macro (`\colvec`) can produce vectors of variable lengths, as

$$\boldsymbol{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \tag{2}$$

- Matrices can be created with the same command. The & switches to the next column:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \end{bmatrix} \tag{3}$$

- Determinants. We provide a simple macro (`\matdet`) whose argument is just a matrix array:

$$\begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 2 & 2 & 2 \end{vmatrix} \tag{4}$$

- If you do longer manipulations, please explain what you are doing: Try to avoid sequences of equations without text breaking up. Here is an example: We consider

$$U_1 = [\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}] \subset \mathbb{R}^4, \quad U_2 = [\begin{bmatrix} -1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}] \subset \mathbb{R}^4. \tag{5}$$

  To find a basis of $U_1 \cap U_2$, we need to find all $\boldsymbol{x} \in V$ that can be represented as linear combinations of the basis vectors of U_1 and U_2, i.e.,

$$\sum_{i=1}^{3} \lambda_i \boldsymbol{b}_i = \boldsymbol{x} = \sum_{j=1}^{2} \psi_j \boldsymbol{c}_j, \tag{6}$$

where $\boldsymbol{b}_i$ and $\boldsymbol{c}_j$ are the basis vectors of U_1 and U_2, respectively. The matrix $\boldsymbol{A} = [\boldsymbol{b}_1|\boldsymbol{b}_2|\boldsymbol{b}_3|-\boldsymbol{c}_1|-\boldsymbol{c}_2]$ is given as

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}. \tag{7}$$

By using Gaussian elimination, we determine the corresponding reduced row echelon form

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{8}$$

We keep in mind that we are interested in finding $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$ and/or $\psi_1, \psi_2 \in \mathbb{R}$ with

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \psi_1 \\ \psi_2 \end{bmatrix} = \boldsymbol{0}. \tag{9}$$

From here, we can immediately see that $\psi_2 = 0$ and $\psi_1 \in \mathbb{R}$ is a free variable since it corresponds to a non-pivot column, and our solution is

$$U_1 \cap U_2 = \psi_1 \boldsymbol{c}_1 = [\begin{bmatrix} -1 \\ 1 \\ 2 \\ 0 \end{bmatrix}], \quad \psi_1 \in \mathbb{R}. \tag{10}$$

2.3 Gaussian elimination

We provide a template for Gaussian elimination. It is not perfect, but it may be useful:

$$\begin{aligned} & \left[\begin{array}{ccccc|c} 1 & -2 & 1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & -3 & 2 \\ 0 & 0 & 0 & -3 & 6 & -3 \\ 0 & 0 & -1 & -2 & 3 & a \end{array}\right] \begin{matrix} \\ \\ \\ -R_2 \end{matrix} \\ \rightsquigarrow & \left[\begin{array}{ccccc|c} 1 & -2 & 1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & -3 & 2 \\ 0 & 0 & 0 & -3 & 6 & -3 \\ 0 & 0 & 0 & -3 & 6 & a-2 \end{array}\right] \begin{matrix} \\ \\ \\ -R_3 \end{matrix} \end{aligned}$$

$$\rightsquigarrow \left[\begin{array}{ccccc|c} 1 & -2 & 1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & -3 & 2 \\ 0 & 0 & 0 & -3 & 6 & -3 \\ 0 & 0 & 0 & 0 & 0 & a+1 \end{array}\right] \begin{array}{l} \\ \cdot(-1) \\ \cdot(-\frac{1}{3}) \\ \\ \end{array}$$

$$\rightsquigarrow \left[\begin{array}{ccccc|c} 1 & -2 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 3 & -2 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & a+1 \end{array}\right]$$

The arguments of this environment are:

1. Number of columns (in the augmented matrix)
2. Number of free variables (equals the number of columns after which the vertical line is drawn)
3. Column width
4. Stretch factor, which can stretch the rows further apart.

3 Answer Template

1) Discrete models

   c)
   d)
   e)

2) Differentiation

   a)
   b)
   d)
   e)

3) Continuous Models

   a)
   b)
   c)
   d)
   e)
   f)
   g)

4) Linear Regression

   a)
   b)
   c)
   d)

5) Ridge Regression

   a)
   b)
   c) i)
      ii)

6) Bayesian Linear Regression

   b)
   c)
   d)
   e) (bonus)